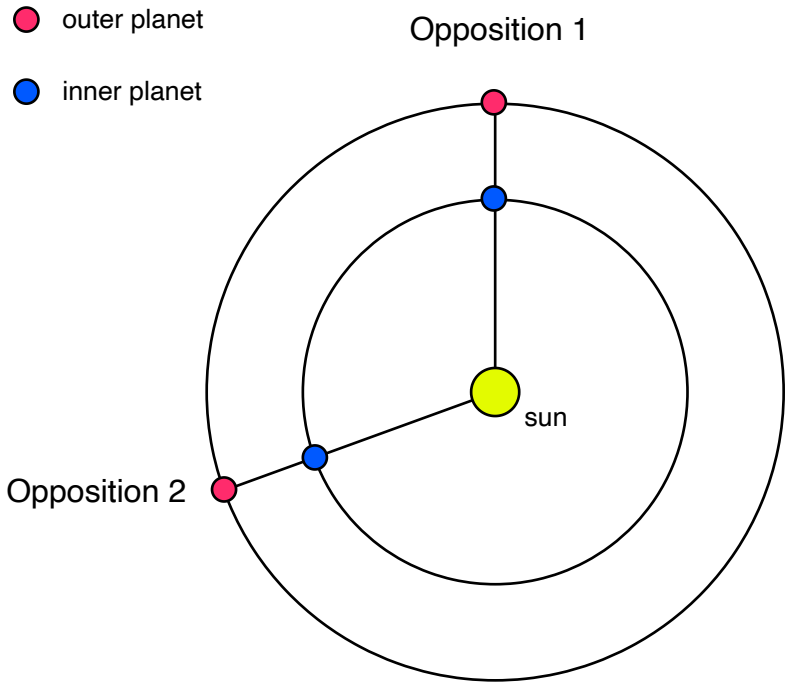
NAME: KEY

Finding the Sidereal Period

The *synodic* period is the time it takes for the earth, sun and planet to achieve the same relative positions to each other. The *sidereal* period is the time it takes a planet to go around the sun once. It is very easy to measure the synodic period directly from earth; it is impossible to measure the sidereal. However, it is very easy to calculate the sidereal period from the synodic.

A planet is in *opposition* when it is exactly opposite the sun; one can draw a straight line from the sun through the earth and then through the planet. The time it takes for a planet to go from opposition to opposition is the synodic period of that planet. Because an inner planet always goes faster than an outer planet, the synodic period is simply the time it takes the inner planet to lap the outer planet, which means the inner planet has gone around the sun *exactly* one more time than the outer planet.



1. A warmup question first: In 2.5 years, how many orbits around the sun does the earth make?

   2.5! (1 year is the time to make exactly 1 orbit)

2. For inner planets, derive an equation that equates the sidereal period (T) with the synodic period (S). (This is easier if you use years instead of days.)

   $T = \frac{\text{time}}{\#\text{ orbits}}$

   In S years, the earth will make S orbits. Since the inner planet is lapping the earth, the inner planet makes exactly S+1 orbits

   So $T = \frac{\text{time}}{\text{orbit}} = \frac{S}{S+1}$

   $\boxed{T = \frac{S}{S+1}}$ but S must be in years!

3. For outer planets, derive an equation that equates the sidereal period (T) with the synodic period (S).

   As above, but now the earth is lapping the outer planet, so the outer planet makes S-1 orbits in S years.

   So $T = \frac{\text{time}}{\text{orbit}} = \frac{S}{S-1}$

   $\boxed{T = \frac{S}{S-1}}$ S in years!

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4. Using your formulas, calculate the sidereal periods for the following planets:

	Mercury	Venus	Mars	Jupiter	Saturn
Synodic Period (days)	116	584	780	399	378
Sidereal Period (days)	88	225	687	4331	10,750
Sidereal Period (years)	0.241	0.615	1.88	11.9	29.4

Mercury

$S = \frac{116 \text{ days}}{365.24} = 0.318 \text{ yrs}$

$T = \frac{S}{S+1} = \frac{.318}{1.318} = 0.241 \text{ yrs}$

$(= 88 \text{ days})$

Mars

$S = \frac{780 \text{ days}}{365.24} = 2.14 \text{ yrs}$

$T = \frac{S}{S-1} = \frac{2.14}{1.14} = 1.88 \text{ yrs}$

$(= 687 \text{ days})$

5. If a superior planet had a synodic period equal to its sidereal period, what would this period be? Which planet most closely approximates this condition?

Superior, so $T = \frac{S}{S-1}$

If $T=S$, then $T = \frac{T}{T-1}$ $\quad 1 = \frac{1}{T-1}$ $\quad T-1 = 1$ $\quad \boxed{T = 2 \text{ yrs}}$

Mars!

6. What would be the sidereal period of an inferior planet that appeared at an inferior conjunction exactly once a year? What if it was a superior conjunction?

Inferior, so $T = \frac{S}{S+1}$

If $S=1$, then $T = \frac{1}{1+1}$ $\quad \boxed{T = 0.5 \text{ yrs}}$ (same answer)

7. Can a superior planet have a synodic period less than one year?

Nope! The earth has lap the outer planet, so it has to go around the sun at least once.

Answers:

1) 2.5 2) $T = \frac{S}{S+1}$ 3) $T = \frac{S}{S-1}$ 4) google it 5) 2 yrs; Mars 6) 0.5 yrs; same 7) no